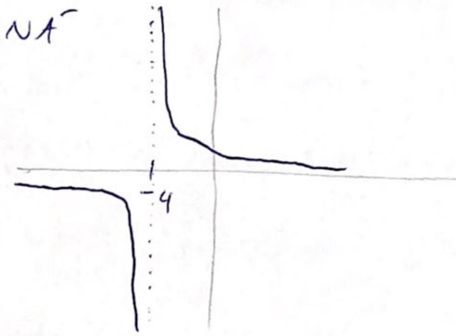
všimnu si: S = [-4,0]

Př.1 $f(x) = \frac{x-4}{x^2-16}$

$$f(x) = \frac{\cancel{x-4}}{(\cancel{x-4})(x+4)} = \frac{1}{x+4}$$

$$f(x) = \frac{x-4}{(x-4)(x+4)}$$

ODSTRANITELNÁ NESPOJITOST



$D_f = \{\pm 4\} / \mathbb{R}$

$= (-\infty,-4) \cup (-4,4) \cup (4,+\infty)$

$$\lim_{x \to +\infty} \frac{x-4}{(x-4)(x+4)} \overset{(F1)}{=} \lim_{x \to +\infty} \frac{x\left(1-\frac{4}{x}\right)}{x\left(x-\frac{16}{x}\right)} \xrightarrow{x \to +\infty} 0$$

$$\lim_{x \to -\infty} \frac{x-4}{x^2-16} \overset{(F1)}{=} \lim_{x \to -\infty} \frac{1-\frac{4}{x}}{x-\frac{16}{x}} \xrightarrow{x \to +\infty} 0$$

$$\lim_{x \to -4^-} \frac{x-4}{x^2-16} = \lim_{x \to -4^-} \frac{\cancel{x-4}}{(x+4)(\cancel{x-4})} = \lim_{x \to -4^-} \frac{1}{x+4} = \frac{1}{0^-} = -\infty$$

$$\lim_{x \to -4^+} \frac{x-4}{x^2-16} = \lim_{x \to -4^+} \frac{\cancel{x-4}}{(x+4)(\cancel{x-4})} = \lim_{x \to -4^+} \frac{1}{x+4} = \frac{1}{0^+} = +\infty$$

$$\lim_{x \to +4^-} \frac{(x-4)}{(x-4)(x+4)} = \lim_{x \to +4^-} \frac{1}{x+4} = \frac{1}{8}$$

$$\lim_{x \to +4^+} \frac{x-4}{(x-4)(x+4)} = \lim_{x \to +4^+} \frac{1}{x+4} = \frac{1}{8}$$

dodefinujeme limitou

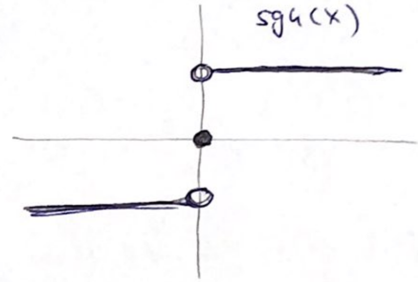
Protože $\lim_{x \to +4^-} = \lim_{x \to +4^+}$.

Př. 2 $\quad f(x) = \dfrac{\sqrt{x^2+2x+5}}{2-2x}$

Funkce signum:

$$\operatorname{sgn}(x) = \begin{cases} 0 & x = 0 \\ 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

$$\operatorname{sgn}(x) = \frac{x}{|x|}$$



Diskriminant výrazu x^2+2x+5:

$$D = b^2 - 4ac = (2)^2 - 4\cdot 5\cdot 1 = -16 < 0$$

výraz pod odmocninou je kladný!.

$D_f = \mathbb{R}\setminus\{1\} \longrightarrow \lim\limits_{x\to+\infty} f(x) \quad \lim\limits_{x\to 1^+} f(x)$

$\lim\limits_{x\to-\infty} f(x) \quad \lim\limits_{x\to 1^-} f(x)$

$$\lim_{x\to+\infty}\frac{\sqrt{x^2+2x+5}}{2-2x} \overset{(\ast 1)}{=} \lim_{x\to+\infty}\frac{\sqrt{x^2\left(1+\frac{2}{x}+\frac{5}{x^2}\right)}}{2-2x} = \lim_{x\to+\infty}\frac{x\sqrt{1+\frac{2}{x}+\frac{5}{x^2}}}{x\left(\frac{2}{x}-2\right)} =$$

$$= \lim_{x\to+\infty}\frac{\sqrt{1+\frac{2}{x}+\frac{5}{x^2}}}{-2+\frac{2}{x}} \overset{\text{VOAL}}{=} \frac{\lim\limits_{x\to+\infty}\sqrt{1+\frac{2}{x}+\frac{5}{x^2}}}{\lim\limits_{x\to+\infty}\left(-2+\frac{2}{x}\right)} = -\frac{1}{2}$$

($\frac{2}{x}\to 0$, $\frac{5}{x^2}\to 0$, $\frac{2}{x}\to 0$)

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+2x+5}}{2-2x} \overset{(\ast 1)}{=} \lim_{x\to+\infty}\frac{\sqrt{x^2\left(1+\frac{2}{x}+\frac{5}{x^2}\right)}}{2-2x} = \lim_{x\to+\infty}\frac{|x|\sqrt{1+\frac{2}{x}+\frac{5}{x^2}}}{x\left(-2+\frac{2}{x}\right)}$$

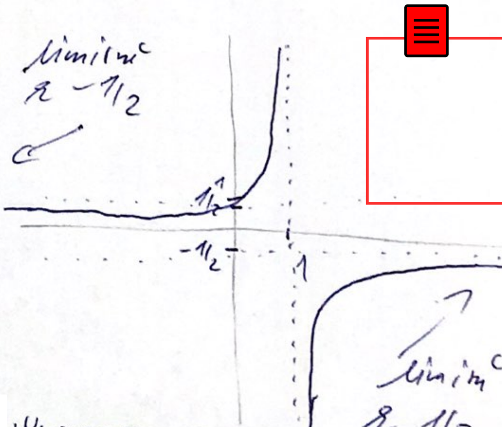
$$= \lim_{x\to-\infty}\frac{1}{\operatorname{sgn}(x)}\,\frac{\sqrt{1+\frac{2}{x}+\frac{5}{x^2}}}{-2+\frac{2}{x}} \overset{\text{VOAL}}{=} \lim_{x\to-\infty}\frac{1}{\operatorname{sgn}(x)}\,\lim_{x\to-\infty}\frac{\sqrt{1+\frac{2}{x}+\frac{5}{x^2}}}{-2+\frac{2}{x}} =$$

$$= (-1)\cdot\left(-\frac{1}{2}\right) = \frac{1}{2}$$

$$\operatorname{sgn}(x) = \frac{x}{|x|}$$

$$\lim_{x\to 1^+}\frac{\sqrt{x^2+2x+5}}{2-2x} = \frac{\sqrt{1^2+2\cdot 1+5}}{0^-} = -\infty$$

$$\lim_{x\to 1^-}\frac{\sqrt{x^2+2x+5}}{2-2x} = \frac{\sqrt{1^2+2\cdot 1+5}}{0^+} = +\infty$$



VYKRESLETE SI V DESMOSU

Musíte být opatrnější při práci s nekonečny. Zejména pak, když vytýkáte z odmocniny, zodpovězte si otázku, proč platí následující (připomeňte si definici absolutní hodnoty):

$$\lim_{x\to-\infty}\sqrt{x^2} = \lim_{x\to-\infty}|x| = +\infty\,.$$